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THE MEASUREMENT OF CONDUCTIVITY IN SEMICONDUCTORS WITH THE AID OF MICROWAVES

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INTRODUCTION

If one measures the dielectric losses in a semiconductor with the aid of microwaves, one has an opportunity to determine the conductivity of the substance in question at those frequencies. The method described in this paper is applicable to thin, disc-shaped samples of substances having a conductivity between $0,1 \cdots 200\ [\text{ohms cm}]^{-1}$. The procedure is based on Drude-Zener's [1] theory and on a measuring method reported by Horner [2].

This method presents the advantage that no contacts are required between the sample substance and the measuring equipment, as is the case with the various D.C. methods. In the present instance a cavity resonator is employed for the determination of the requisite values of measurement [3].

1. THE RESONATOR METHOD

The disc-shaped sample is inserted in a cavity resonator in accordance with Fig. 1. The resonator oscillates in a TM_{010} mode. One may then write for the magnetic and electric field.

$$(1) \qquad \begin{cases} H_o = A\, J_1 (kr)\ e^{j\omega t} \\ E_o = \dfrac{k}{\sigma + j\omega\varepsilon}\ A\, J_o (kr)\ e^{j\omega t} \end{cases}$$

where

$k^2 = -j\omega\mu\ (\sigma + j\omega\varepsilon)$
A = a constant
$\omega = 2\pi f$
μ = the permeability
ε = the dielectric constant
σ = the conductivity
J_o, J_1 = Bessel's functions of the first order

The energy accumulated in a resonator is independent of time and it can be calculated either from the magnetic or from the electric field by the formula

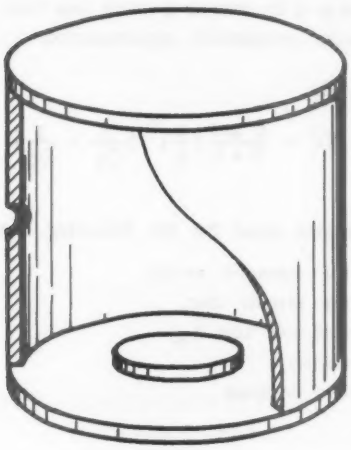

Fig. 1.

$$W = \iiint \tfrac{1}{2}\,\mu\, H^2\, dv = \iiint \tfrac{1}{2}\,\varepsilon\, E^2\, dv$$

where dv is a volume element and the integral extends over the entire resonator.

The power dissipation in the semiconductor sample can be calculated in the same manner by the formula

$$(2) \qquad P_s = \iiint \tfrac{1}{2}\,\sigma\, E^2\, dv = A^2\, \frac{\pi\,\mu\,\omega\, l\, b^2}{2}\, \left[J_o^2\left(\frac{2\pi}{\lambda_o} b\right) + J_1^2\left(\frac{2\pi}{\lambda_o} b\right)\right] \operatorname{tg}\delta$$

According to Horner [2], this formula can be written for the loss angle

$$(3) \qquad \operatorname{tg}\delta = \frac{(a/b)^2 + F\chi}{(\chi + 1)\, F^2 \left\{1 + \left[J_1^2\left(\frac{2\pi}{\lambda_o} b\right) / J_o^2\left(\frac{2\pi}{\lambda_o} b\right)\right]\right\}} \left[\frac{1}{Q_1} - \frac{1}{Q_2}\right]$$

in the case that the thickness of the sample is much less than the total length of the resonator, this renders with permissible approximation

$$\text{(4)} \qquad \operatorname{tg}\delta = \frac{0.269}{\chi + 1} \left(\frac{a}{b}\right)^2 \left[\frac{1}{Q_1} - \frac{1}{Q_2}\right]$$

the symbols in the equations stand for the following quantities:

- a = radius of the resonator cavity
- b = radius of the sample disc.
- F = a function of b/a and λ_o
- μ = the permeability
- ε = the dielectric constant
- $\chi = \frac{\varepsilon}{\varepsilon_o} - 1$
- Q_1 = the Q value for the resonator with the semiconductor sample
- Q_2 = the Q value for the resonator with a sample having the same dimensions and dielectric constant as the semiconductor sample, but free of losses

Q_2 can be determined by the formula

$$\text{(5)} \qquad Q_2 = \frac{1}{d} \frac{(a/b)^2 + F^2 \chi}{a\,(a + b)\,/b^2 + F^2 \chi}$$

The value of F is obtained in accordance with Horner [2] and χ can be determined experimentally in accordance with [3], while the value of d is obtained from the formula

$$\text{(6)} \qquad d = \frac{a\,l}{Q_o\,(a + 1)}$$

where

Q_o = the Q value for the empty resonator.

2. THE COMPLEX CONDUCTIVITY σ_c AND THE LOSS ANGLE tg δ

In accordance with Drude-Zener's theory, one may write for the complex conductivity:

$$\sigma_c(\omega) = \frac{4\pi ne^2}{3kT}\left(\frac{m^*}{2\pi kT}\right)^{\frac{3}{2}} \int_0^\infty \frac{\tau v^4}{1+i\omega\tau} e^{-\frac{m^* v^2}{2kT}}\, dv \tag{7}$$

where

n = carrier density
e = carrier charge
k = Boltzmann constant
T = absolute temperature
m^* = carrier effective mass

It is customary to introduce a new variable u, which is analogous to $\omega\tau$. Let us choose

$$u = \frac{3\pi^{\frac{1}{2}}}{4\varrho_o}\left(\frac{\omega m^*}{ne^2}\right) \tag{8}$$

However, two scattering processes have to be taken into account in semiconductors. One of them is derived from the lattice vibration, i. e., a thermal scattering, while the other one has its source in an impurity scattering, Donovan [1]. For the free path one may write under these circumstances

$$\frac{1}{l} = \frac{1}{l_t} + \frac{1}{l_i} \tag{9}$$

where l_t is the free path in the case of thermal scattering and l_i that in the case of impurity scattering. For the effective relaxation time $\tau = l/v$, where v is the electron velocity, we find the expression

$$\tau = \frac{\pi^{\frac{1}{2}} m^* x^3 \zeta}{8 n e^2 \varrho_i (\zeta + x^2)} \tag{10}$$

where $x = m^* v^2/kT$ and $\zeta = 6\ \varrho_i/\varrho_o$. Equation (7) is now transformed into

$$\sigma_c(\omega) = \frac{4 n e^2 \varepsilon_o}{3 \pi^{\frac{1}{2}} m^*} \int_0^\infty \frac{\tau (1 - i\omega\tau)}{1 + (\omega\tau)^2} x^{\frac{3}{2}} e^{-x}\, dx \tag{11}$$

which yields a real component

$$\sigma_r(\omega) = \frac{\varepsilon_o}{6 \varrho_i} \int_0^\infty \frac{x^3 (1 + \frac{x^2}{\zeta}) e^{-x}\, dx}{(1 + \frac{x^2}{\zeta})^2 + \mu_i x^3/36} \tag{12}$$

and an imaginary component

$$\sigma_i(\omega) = \frac{-\mu_i \varepsilon_o}{\varrho_i} \int_o^\infty \frac{x^{\frac{9}{2}} e^{-x}\, dx}{36\left(1 + \frac{x^2}{\zeta}\right)^2 + \mu_i^2 x^3} \tag{13}$$

Here, u_i is the expression for u in semiconductors where impurity scattering occurs and u_t the u value for thermal scattering. Both u values are mutually related as follows:

$$\frac{1}{\mu_t} + \frac{1}{\mu_i} = \frac{G(\zeta)}{\mu} \tag{14}$$

On the strength of the foregoing the loss angle may be written

$$\operatorname{tg}\delta = \frac{\alpha}{\beta + \omega \varepsilon \varepsilon_o / \sigma_o} \tag{15}$$

where

$\alpha = \sigma_r / \sigma_o$

$\beta = \sigma_i / \sigma_o$

ε_r = the dielectric constant of the sample

$\varepsilon_o = 8,854\ 10^{-12}$ Farads/m

σ_o = the conductivity of the sample

3. EXPERIMENTAL MEASUREMENTS

The measurements have been carried out with a microwave set-up in the 3 cm band. Signal generator: HP 620 A. The klystron was modulated by a 1 kc square wave. The output signal from the detector was amplified in the HP 415 A indicator.

The object of measurement employed in the experiments was a single crystal silicon sample of p type and having a resistivity of 10 ohms · cm. The thickness of the sample was h = 0,72 mm and its diameter 2 b = 10,87 mm.

Resonator dimensions: Length 1 = 26,6 mm; diameter 2 a = 24,2 mm.

Experimental values:

$$Q_0 = 3630 \qquad Q_2 = 2670$$

$$Q_1 = 750 \qquad d = 2{,}5\ 10^{-3}\ \text{mm}$$

With these values, we find for σ_0 the value, according to (15).

$$\sigma_0 = 0{,}103\ [\text{ohms cm}]^{-1}, \text{ or } \varrho = 9{,}6\ \text{ohms cm}$$

The requisite data relating to mobility and effective mass were taken from earlier measurements with the same sample [3].

It is obvious from the foregoing that Q_2 cannot be measured experimentally because it is difficult to find a substance having the same dielectric constant as the semiconductor sample but no losses. It was found in measurements with sulphur that the Q value decreased by 35 to 45 % of the Q_0 value. Horner [2] reports deviations amounting to 30 %.

4. SUMMARY

A method has been presented for the determination of the conductivity of semiconductors at microwave frequencies. It is found that the measurements can be carried out without any contact problems, which always arise in connection with D. C. measurements.

In the calculation of the ultimate value of the conductivity those scattering processes have been taken into account which occur owing to the presence of impurities and lattice vibrations. The error limits come up to 10 % and they are dependent on the losses in the sample because accurate measurements are difficult to carry out at high and at low Q values.

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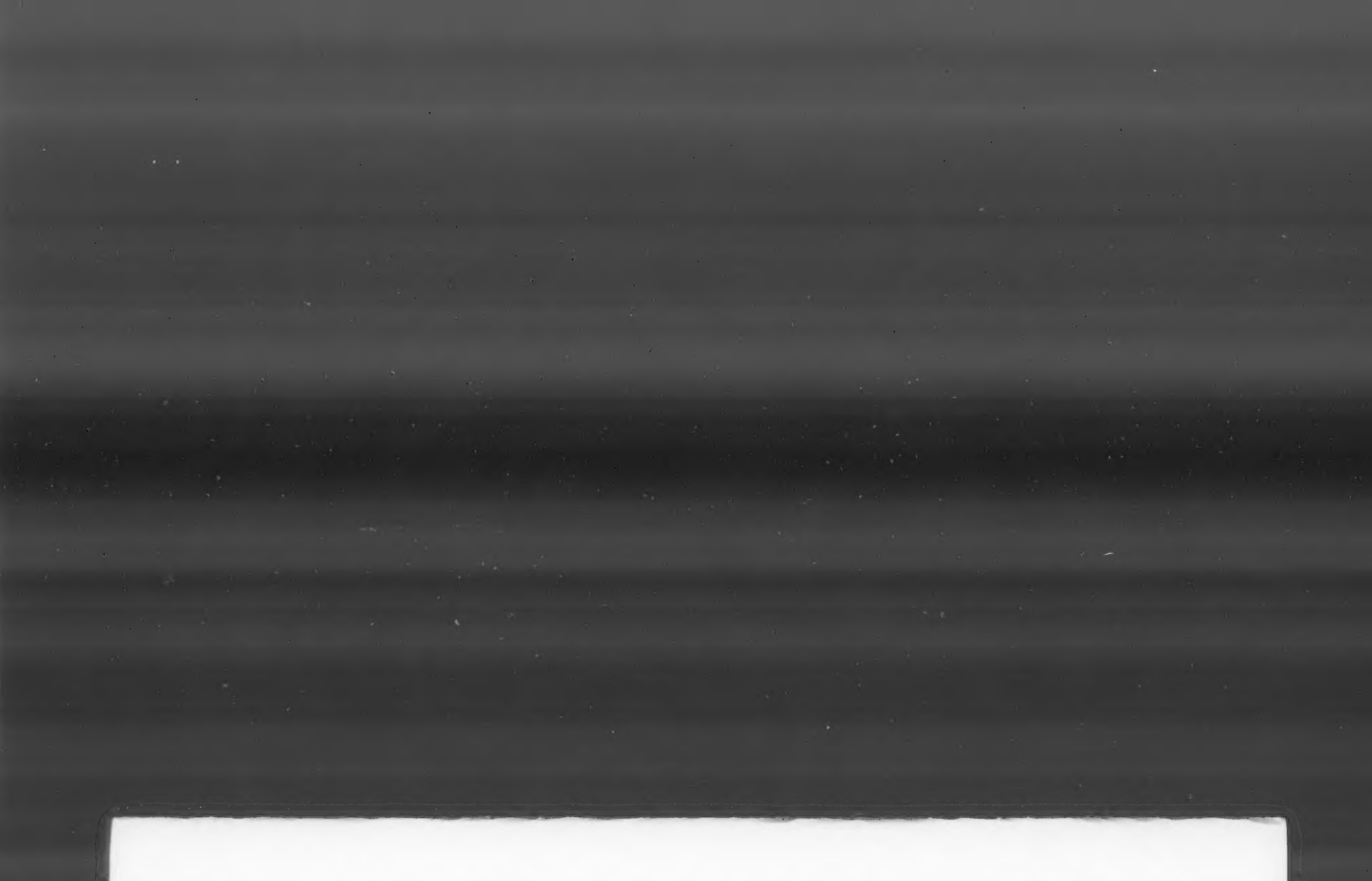
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